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MODAL DECOMPOSITION OF COVARIANCE SEQUENCES FOR PARAMETRIC SPEC--ETC(U)
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Proceedings ICASSP, Atlanta

March 1981

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MODAL DECOMPOSITION OF COVARIANCE SEQUENCES
FOR PARAMETRIC SPECTRUM ANALYSIS¹

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ABSTRACT

In this paper we make the point that a wide variety of spectrum types admit to modal analysis wherein the modes are characterized by amplitudes, frequencies, and damping factors. The associated modal decomposition is appropriate for both continuous and discrete components of the spectrum. The domain of attraction for the decomposition includes ARMA sequences, harmonically- or nonharmonically-related sinusoids, damped sinusoids, white noise, and linear combinations of these.

Numerical results are presented to illustrate the identification of mode parameters and corresponding spectra from finite records of perfect and estimated covariance sequences. The results for sinusoids and sinusoids in white noise are interpreted in terms of inphase and quadrature effects attributable to the finite record length.

INTRODUCTION

This is a paper about parametric covariance sequence approximation for the purposes of model identification and spectrum analysis. As with all parametric methods of analysis the key idea is to pose a defensible model for the data, or some important statistical descriptor such as the covariance sequence, and then to identify model parameters.

Our approach is to represent the covariance sequence of a wide-sense stationary process in a modal decomposition. Amplitudes, frequencies (formants), and damping constants then become mode parameters to be identified. These mode parameters may be used in vibration analysis, speech analysis, and forecasting and control.

Corresponding to the modal decomposition for the covariance sequence is a modal decomposition for the underlying process. Thus the decomposition becomes an analysis/synthesis tool applicable to data-compressed speech and data communication, and to stochastic simulation. The resulting synthesizer might be termed a statistical vocoder to

distinguish it from source and channel vocoders, and to suggest that identification of formants and other mode parameters proceeds statistically.

The domain of attraction for our modal decomposition of the covariance sequence includes ARMA processes, harmonically- or nonharmonically-related sinusoids, white noise, and linear combinations of these. This means the decomposition generalizes the covariance sequence model implicit in DFT, Pisarenko [3], and linear prediction formulations of the parametric spectrum analysis problem in the same way that Prony's ancient model [1] generalizes DFT, almost periodic, and autoregressive models for data.

COVARIANCE SEQUENCE APPROXIMANT

Motivated by the form of the covariance sequence for line and rational spectra, we propose the following order- p covariance sequence approximant:

$$r_t(p) = \sum_{i=1}^p A_i z_i^t, \quad t=0,1,2,\dots$$

$$r_{-t}(p) = r_t(p)$$

$$z_i = \rho_i e^{j\omega_i}$$

We call z_i^t the i th complex mode, with frequency ω_i and radius ρ_i , and A_i the corresponding mode weight. The weights and modes appear in complex conjugate pairs. Typically a subset of the ρ_i are unity to model the discrete component of the spectrum, and the rest are less than unity to model the continuous part of the spectrum. See [2] for review of discrete-time spectral theory.

The Prony Device: Begin with a finite covariance string $(r_0, r_1, \dots, r_{p-1}, \dots, r_{2p-1})$, assumed to be the first $2p$ Fourier series coefficients for a density $f(\omega)$. Solve for the regression coefficients (a_1, a_2, \dots, a_p) that satisfy

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$$\begin{bmatrix} r_{p-1} & r_{p-2} & \dots & r_0 \\ r_p & & & \\ & & & \\ & & & r_{p-2} \\ r_{2p-2} & r_p & r_{p-1} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_p \end{bmatrix} = - \begin{bmatrix} r_p \\ r_{p+1} \\ \vdots \\ r_{2p-1} \end{bmatrix}$$

Form the polynomial

$$P(z) = 1 + a_1 z^{-1} + \dots + a_p z^{-p}$$

and find its roots z_1, z_2, \dots, z_p . Construct the Vandermonde matrix

$$V = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & & z_p \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{p-1} & z_2^{p-1} & \dots & z_p^{p-1} \end{bmatrix}$$

and solve for weights A_1, A_2, \dots, A_p in the following system of equations:

$$V \begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_p \end{bmatrix} = \begin{bmatrix} r_0 \\ r_1 \\ \vdots \\ r_{p-1} \end{bmatrix}$$

The modal decomposition

$$r_t(p) = \sum_{i=1}^p A_i z_i^t$$

reconstructs (r_0, \dots, r_{2p-1}) and extends it as a linear combination of complex modes. If (r_0, \dots, r_{2p-1}) , $p > p_0$, comes from an ARMA (p_0, p_0) process then $A_i = 0$ for $p > p_0$ and the modal decomposition matches r_t for all t . Here $p_0 < p$.

The Fejer Device: The trigonometric sum

$$\hat{f}(\omega) = \sum_{t=-p}^p r_t(p) \rho^{|t|} e^{j\omega t}$$

uniformly approximates $f(\omega)$ as $p \uparrow$ and $p \downarrow$. Note the term $\rho^{|t|}$ simply draws the modes $z_i^{|t|}$ into $(\rho z_i)^{|t|}$ —corresponding to pole locations more interior to the unit disc.

ANALYSIS AND SYNTHESIS

Assume we are given a modal decomposition of the form

$$r_t(p) = \sum_{i=1}^p A_i z_i^t, \quad t=0,1,\dots$$

The following question naturally arises: is there a finite-dimensional spectrum representation (or vocoder) that generates stochastic realizations of the wide-sense stationary process x_t that has $r_t(p)$ for its covariance? The answer is a qualified yes, and the development goes as follows.

Let X_t be a complex process generated by the autoregressive scheme

$$X_t = \rho X_{t-1} + \epsilon_t$$

where

$$X_0 : N(0, A)$$

$$\epsilon_t : \text{sequence of i.i.d. } N(0, (1-|\rho|^2)A) \text{ random variables}$$

This process has covariance

$$R_t = A \rho^t, \quad t=0,1,\dots$$

The process

$$x_t = X_t e^{j\omega t}$$

with ω fixed and independent of X_t has covariance

$$r_t = A z^t \\ z = \rho e^{j\omega}$$

Think of x_t generated this way as $x_t(A, \rho, \omega)$. Generate p independent processes like this of the form $x_t = x_t(A_i, \rho_i, \omega_i)$ and sum to get

$$x_t = \sum_{i=1}^p x_t^i$$

with covariance

$$r_t = \sum_{i=1}^p A_i z_i^t; \quad z_i = \rho_i e^{j\omega_i}$$

This statistical vocoder is illustrated in Figure 1. It generates a special class of processes with modal decompositions such that $A_i \geq 0$. It captures DFT and Pisarenko decompositions as special cases.

²The notation $X_0 : N(0, A)$ indicates X_0 is complex normal with mean zero and variance $A > 0$. The model is trivially generalized to other distributions.

PARAMETER IDENTIFICATION FROM A FINITE RECORD

In this section we propose a two-step (a-one, a-two) procedure for obtaining a modified least squares fit of $r_t(p)$ to a covariance string that has been estimated as follows:

$$r_t = \frac{1}{N} \sum_{i=1}^N x_i x_{i+t}$$

Least Squares: Define the squared error between $r_t(p)$ and r_t as follows:

$$E = \sum_{t=-N+1}^{N-1} (r_t - r_t(p))^2$$

Setting derivatives of E with respect to (A_i, z_i) , $i=1, 2, \dots, p$ to zero yields a discrete form of the Agrain-Williams equations [6], slightly modified to reflect the fact we are approximating a two-sided sequence:

$$\frac{\partial E}{\partial A_i} : \sum_{t=-N+1}^{N-1} (r_t - r_t(p)) z_i^{|t|} = 0 \quad (i=1, 2, \dots, p)$$

$$\frac{\partial E}{\partial z_i} : \sum_{t=-N+1}^{N-1} (r_t - r_t(p)) A_i |t| z_i^{|t|-1} = 0 \quad (i=1, 2, \dots, p)$$

Using the symmetry of the r_t and $r_t(p)$ sequences we may write this system of equations as

$$P^T Q \underline{r} = P^T Q \underline{A}$$

$$\tilde{P}^T Q \underline{r} = \tilde{P}^T Q \underline{A}$$

where

$$P = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_p \\ \vdots & \vdots & \ddots & \vdots \\ z_1^{p-1} & z_2^{p-1} & \dots & z_p^{p-1} \end{bmatrix}, \quad Q = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 2 & 0 & \dots & 0 \\ 0 & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 2 \end{bmatrix}$$

$$\underline{r}^T = (r_0, r_1, \dots, r_{N-1})$$

$$\underline{A} = (A_1, \dots, A_p)$$

$$\tilde{P} = \sum_{i=1}^p A_i \frac{\partial}{\partial z_i} P$$

This system of equations is nonlinear in the parameters (A_i, z_i) and calls for iterative procedures if the solution is to be "exact". We suggest here a modification of these equations for which a tractable solution procedure exists.

A Modification & Solution: Consider the

modified squared error

$$E_m = \sum_{t=p}^{N-1} \left(\sum_{i=0}^p a_i (r_{t-i} - r_{t-i}(p)) \right)^2 ; a_0 = 1$$

This amounts to filtering the errors with a moving average filter and counting them after p steps have elapsed. If the filter weights are selected so that

$$\sum_{i=0}^p a_i z^{-i} = \pi (1 - z_1 z^{-1})$$

then the approximating $r_t(p)$ sequence satisfies this homogeneous difference equation on its tail:

$$\sum_{i=1}^p a_i r_{t-i}(p) = 0, \quad t \geq p$$

This is one of the things Prony saw while studying the effects of alcohol vapor pressures in 1795.

With this modification the contribution of $r_t(p)$ to the modified squared error is annihilated and we are left with

$$E_m = \sum_{t=p}^{N-1} \left(\sum_{i=0}^p a_i r_{t-i} \right)^2$$

Minimization with respect to the a_i leads to a covariance-method of linear prediction on the tail of the covariance sequence,

$$R^T \underline{R} \underline{a} = -R^T \underline{r}_p$$

where

$$R = \begin{bmatrix} r_{p-1} & \dots & r_1 & r_0 \\ \vdots & \ddots & \vdots & \vdots \\ r_{N-1} & r_{N-2} & \dots & r_{N-p} \end{bmatrix}$$

$$\underline{r}_p^T = (r_p, r_{p+1}, \dots, r_{N-1})$$

$$\underline{a}^T = (a_1, \dots, a_p)$$

The elements of $R^T R$ are "covariances of covariances".

There are a variety of fast algorithms for solving these equations. Once the a_i are found the corresponding modes z_i are obtained using a polynomial rootfinding routine such as Muller's algorithm [4]. With these modes determined we may solve a linear system of equations for the mode weights A_i . This is basically Prony's method at work except for the fact that the a_i are obtained from a squared error criterion rather than Prony's point-wise criterion that results from setting $N=2p$:

$$R_{\hat{x}} = \hat{r}$$

One cannot help wondering what path statistical inference might have taken had Gauss been aware of Prony's parametric interpolation formulae when he published his work on least-squares in 1809. For at this early date all the ingredients of a least-squares theory of rational approximation would have existed.

APPLICATIONS AND NUMERICAL RESULTS

Here we present the results of several numerical experiments. These experiments are summarized in Table 1. In the table the column labels have the following meanings:

Exact Model-Underlying model that generated data

No. Samples - Number of samples used to estimate r_t (= means exact covariance used)

No. Corr. Lags - Number of correlation lags used in fitting algorithm

Fitted ARMA(.,.) - For the modal decomposition the initial order denotes the initial number of poles identified. When the final order differs from the initial order this means a noise pole (at $z=0$) was added. For the modified least squares (MLS) the initial order tells what order AR was fit to initialize. The final order gives the approximating ARMA.

Two Closely-Spaced Sines in WGN: Here

$$r_t = \cos \frac{\pi}{4} t + 0.01 \cos .95 \frac{\pi}{4} t + 100 \delta_t$$

See Figure 2 for spectrum of approximating modal decomposition.

Sine in AR Noise: Here

$$r_t = \cos t + z^{-1} \{H(z)H(z^{-1})\}$$

$$H(z) = 1/(1-j0.8z^{-1})(1+j0.8z^{-1})$$

See Figure 3 for spectrum of approximating modal decomposition.

Comment: In a 1976 paper on digital filter design [6] one of us advocated a two step procedure for fitting long AR sequences to exact correlation sequences, followed by modified least squares fitting of an ARMA(p,q). The modified least squares procedure was originally proposed by Kalman and subsequently fully developed by Mullis and Roberts [7]. At the end of that paper we suggested that the same procedure might be applied to ARMA spectrum analysis. This suggestion we have explored in conjunction with our modal decomposition studies. Spectral results for modified least squares fitting from long AR models are shown together with spectra for modal decompositions in the following examples.

Sine: Here

$$x_t = \sin \frac{\pi}{4} t$$

$$r_t = \frac{1}{100} \sum_{i=1}^{100} x_i x_{i+t}$$

See Figure 4 for spectra of approximating modal decomposition and approximating modified least squares fit.

Two Closely-Spaced Sines: Here

$$x_t = \sin \frac{\pi}{4} t + \cos .95 \frac{\pi}{4} t$$

$$r_t = \frac{1}{160} \sum_{i=1}^{160} x_i x_{i+t}$$

See Figure 5 for spectra of approximating modal decomposition and approximating modified least squares fit.

ARMA(1,1): Here x_t is the output of the following filter excited by white noise:

$$H(z) = \frac{1-0.95z^{-1}}{1-0.25z^{-1}}$$

See Figure 6 for spectra of approximating modal decomposition and approximating modified least squares fit.

ARMA(3,2): Here x_t is the output of the following filter excited by white noise:

$$H(z) = \frac{1-1.75z^{-1}+0.8z^{-2}}{1-1.5z^{-1}+1.21z^{-2}-0.455z^{-3}}$$

See Figure 7 for spectra of approximating modal decomposition and approximating modified least squares fit.

More examples may be found in [5].

CONCLUSIONS

We have proposed an approach to covariance sequences and rational spectrum approximation that captures discrete spectra and ARMA spectra as special cases. The approach is based on a modal decomposition for the covariance sequence wherein the modes correspond to poles. For a special class of processes with modal decomposition there is a stochastic synthesis algorithm (or statistical vocoder) that may be used to generate realizations. This could be a valuable algorithm for data-compressed communication based on the analysis/synthesis ideas of this paper.

Numerical results for noise-free and noisy sinusoids are encouraging. A virtue of this technique-and a virtue we want to emphasize-is that one obtains from the analysis technique both a spectrum and a map of the underlying modes of the covariance sequence. These modes may be more valuable in some instances than the spectrum itself. In fact, the spectrum $f(\omega)$ can often obscure very interesting fine structure in the data that one can observe directly in the covariance sequence approximant $r_t(p)$.

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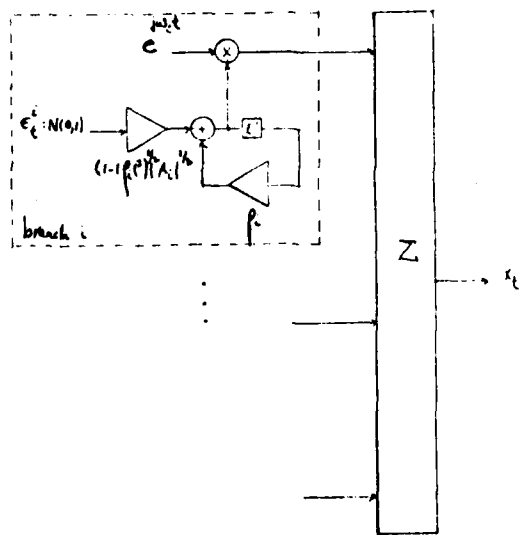


Figure 1. Statistical Vocoder

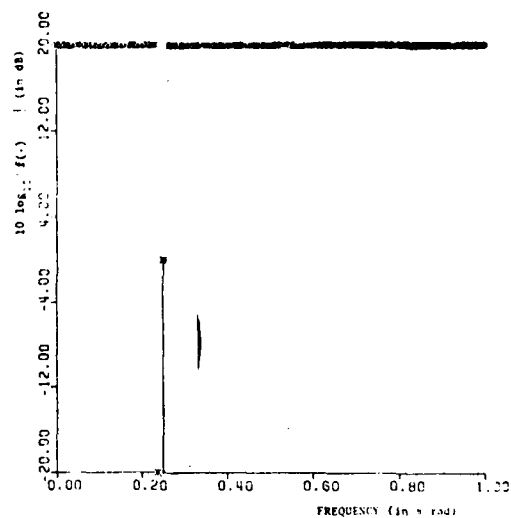


Figure 2. Spectral estimate for 2 sines in white noise. Given 15 known covariance lags. SNR = -20 dB, -40 dB

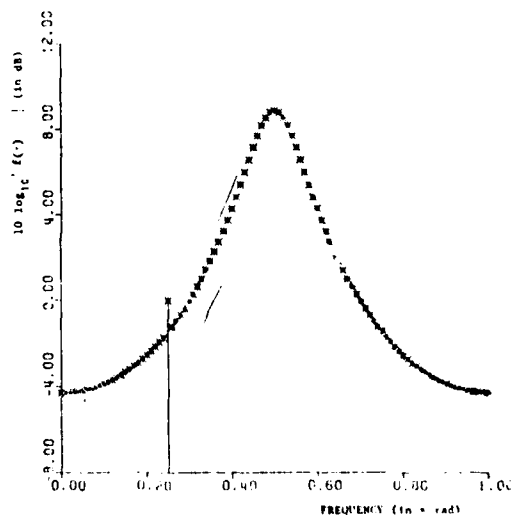


Figure 3. Spectral estimate for sine in second order autoregressive noise. Given 20 known covariance lags

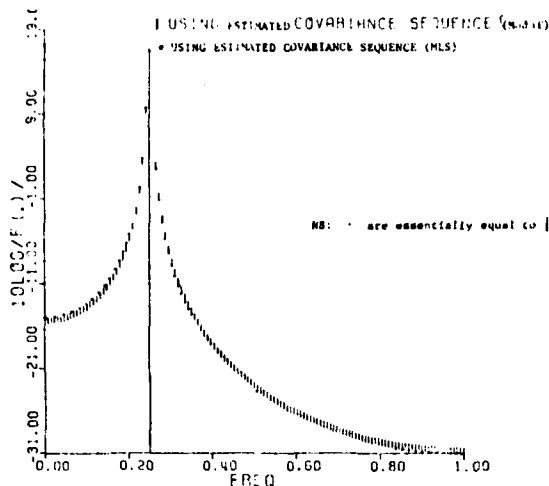


Figure 4. Spectral estimate for sine. Using 20 bias-estimated lags from 100 samples

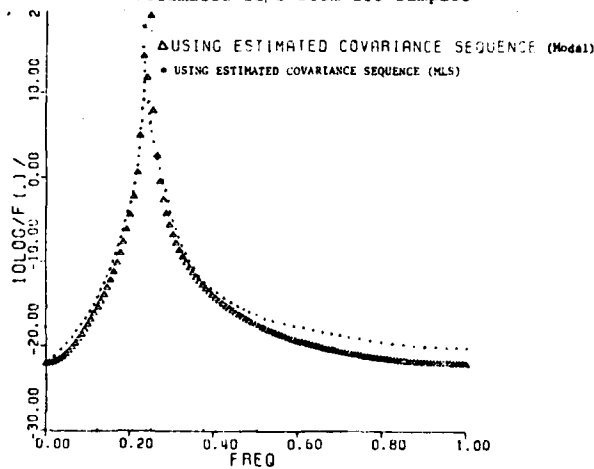


Figure 5. Spectral estimate for two closely-spaced sines. Using 20 bias-estimated covariance lags from 160 samples

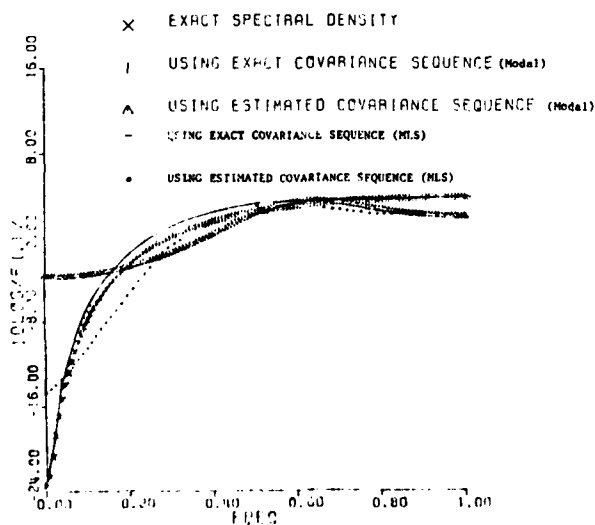


Figure 6. Spectral estimates for ARMA(1,1). Using 40 bias-estimated covariance lags from 200 samples

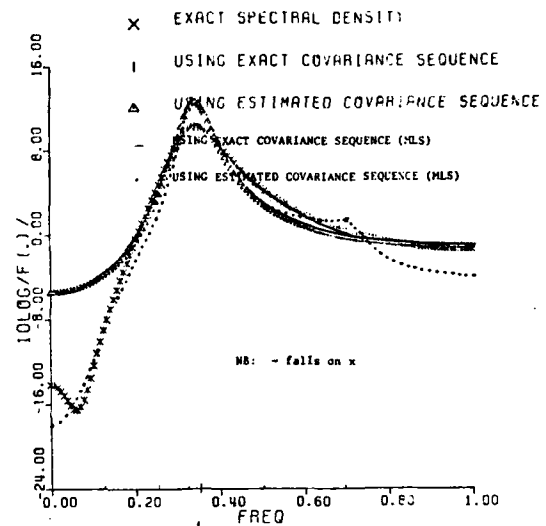


Figure 7. Spectral estimate for ARMA(3,2). Using 40 bias-estimated covariance lags from 200 samples

Exact Model	No. Samples	No. Corr. Lags	Fitted ARMA(·,·)	
			Initial Order	Final Order
1. Two Sines + WGN	∞	16		
a. model			(5,4)	(5,4)
b. mod. least squares (mls)			NC	NC
2. Sine in AR Noise	∞	20		
a. model			(5,4)	(5,4)
b. mls			NC	NC
3. Sine	100	20		
a. model			(4,3)	(5,4)
b. mls			(20,0)	(5,4)
4. Two Sines	160	20		
a. model			(8,7)	(9,8)
b. mls			(20,0)	(5,4)
5. ARMA(1,1)	200	40		
a. model			(2,1)	(3,2)
b. mls			(40,0)	(3,2)
6. ARMA(3,2)	200	40		
a. model			(3,2)	(4,3)
b. mls			(40,0)	(4,3)

Table 1. Experiments. NC means not computed.

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